American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Summer 2009

quiz # 2 - solution part 1

1. Suppose that $P(X=0) = 1 - P(X=1)$. If $E(X) = 3Var(X)$, find $P(X=0)$.

X is a bernouilli random variable. $E(X) = p$ and $Var(X) = p(1-p)$, by equating we find $p = 2/3$; and hence $P(X=0) = 1/3$.

2. The probability of winning a prize on the National lottery is 0.018. Suppose that you decide to play one line per week. Let X be the number of weeks until you first win a prize. Find $P(X \geq 10)$.

$X \rightsquigarrow \mathcal{G}(0.018)$; the event $\{X \geq 10\}$=$\{$event the first nine weeks are all loses$\}$, and hence $P(X \geq 10) = (0.982)^9 = 0.849$

3. A sample of 6 items is selected from a box containing 40 items of which 8 are defective. Find the expected number of defective items in the sample.

$E(X) = 6.\frac{8}{40} = 1.2$, wether the selection is with (binomial) or without (hypergeometric) replacement.

4. Suppose that the number of times during a year that an individual catches a cold can be modeled by a Poisson random variable with an expectation of 4. Further suppose that a new drug based on Vitamin C reduces the expectation to 2 (but is still a Poisson distribution) for 80% of the population, but has no effect on the remaining 20% of the population. Find the probability that an individual taking the drug has 2 colds in a year.

Let $X \rightsquigarrow \mathcal{P}(2)$, and $Y \rightsquigarrow \mathcal{P}(4)$, and let $A =\{$event an individual taking the drug has 2 colds in a year$\}$, then

$P(A) = 0.8P(X=2) + 0.2P(Y=2) = 0.8 * 0.27 + 0.2 * 0.146 = 0.246$ (from Poisson table)

5. The probability of being dealt a full house in a hand of poker is approximately 0.0014. Find the probability that, in a 10 hand of poker hands, you will be dealt at least one full house.

Let X be the number of full houses in the 10 hands; $X \rightsquigarrow b(10, 0.0014)$.

$P(X \geq 1) = 1 - P(X=0) = 1 - (0.9986)^{10} = 0.0139$

6. Two teams A and B play a series of games that ends when one of them has won 3 games. Suppose that each game played is, independently, won by team A with probability p. Let X be the number of games played. Find the pdf of X.

$X \mapsto \{3, 4, 5\}$

k	$P(X=k)$
3	$p^3 + (1-p)^3$
4	$3p^3(1-p) + 3p(1-p)^3$
5	$6p^2(1-p)^2$

7. Each game you play is a win with probability $2/3$. You plan to play 5 games, but if you win the fifth game, then you will keep on playing until you lose. Find the expected number of games that you play.

Let X be the number of games that you play until you lose starting form the fifth game, then X is geometric with $p = 1/3$, and $E(X) = 3$, and you already played 4 games, then the expected number of games played is $4 + 3 = 7$.

8. Let X be a negative binomial random variable $NB(r, p)$, and Y be a binomial random variable $b(n, p)$. Show that $P(X > n) = P(Y < r)$.

$\{X > n\}$=$\{$event that more than n trials are needed to have r success$\}$=$\{$event that there are at most r success in n trials$\}$=$\{Y < r\}$